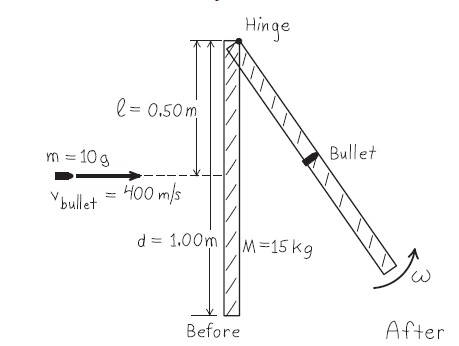
**Homework 5. Solutions**

Teacher: Paul Briard

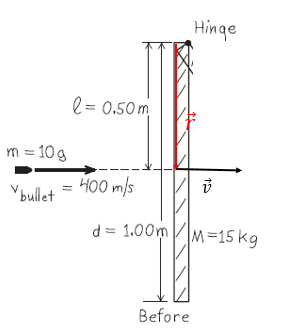
**Ex. 1.** As shown on the figure, a door 1.00 m wide, of mass 15 kg, can rotate freely about a vertical axis through its hinges. A bullet with a mass of 10 g and a speed of strikes the center of the door, in a direction perpendicular to the plane of the door, and embeds itself there. (a) Describe the magnitude of the angular momentum of the bullet (treated as a particle) about the hinge shown on the figure, just before the collision in respect to , the bullet velocity, the bullet mass and the distance shown on the figure. (b) Describe the magnitude of the angular momentum of the door about the hinge just before the collision. (c) We consider the bullet and the door as the system. Calculate the moment of inertia about the axis along the hinges of the system “bullet+door” (you don’t need to demonstrate the moment of inertia of the door, you can use the result of the previous homework). Describe the angular momentum about the hinge of the system “bullet + door” after the impact in respect with and , their angular velocity of rotation (d) No external torque is exerted on the system “bullet+door”. Calculate the value of .

**Hint:** The axis passing through the hinges is not an axis of symmetry of the system “bullet+door”. Usually, in that situation, , the angular momentum about a point of the axis of rotation don’t have the direction of the axis of rotation, when the axis of rotation is not an axis of symmetry of the body. However, in this exercise, before the collision, the angular momentum about the hinge of the system “door+bullet” is directed along the axis passing through the hinges. And because there is not external torque, after the collision, the angular momentum of the system “door+bullet” about the hinge is still directed toward the axis passing through the hinges, which means you can use: to describe the angular momentum of the system after the collision.



**Solution:**

(a) The bullet is treated as a particle. Just before the impact, the bullet, is approximately at a distance from the hinge.



Its angular momentum about the hinge is:

The magnitude of vector is also the distance . Direction of is the axis passing through the hinges, i.e. the axis of the rotation after the collision.

(b) The angular momentum of the door about the hinge just before the collision is zero.

(c) Moment of inertia of the door about the axis passing through the hinges is:

The moment of inertia of the bullet about the axis passing through the hinges is (the bullet is seen as a particle):

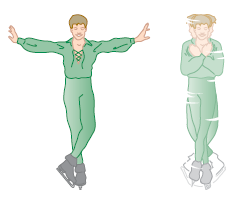
The moment of inertia of the system passing through the hinges is:

The angular momentum of the system “door+bullet” about the hinge after the collision is:

No external forces are exerted (the rotational motion occurs because of the collision), the angular momentum about a point of the system “bullet+door” is constant, we obtain:

**Ex. 2.** The outstretched hands and arms of a figure skater preparing for a spin can be considered a slender rod pivoting about an axis through its center, as shown on the figure. When the skater’s hands and arms are brought in and wrapped around his body to execute the spin, the hands and arms can be considered a thin-walled, hollow cylinder. His hands and arms have a combined mass of 8.0 kg. When outstretched, they span 1.8 m; when wrapped, they form a cylinder of radius 25 cm. The moment of inertia about the rotation axis of the remainder of his body is constant and equal to . (a) Describe the moment of inertia of the hollow cylinder of internal radius and external radius , length , uniform mass density about its axis in respect with and its total mass. Then, describe the moment of inertia of a thin hollow cylinder (of radius ) about its axis in respect with its mass and (b)If his original angular speed is 0.40 (rev means “revolution”, i.e. one turn) what is the final angular speed of the spinning figure skater (in unit )?

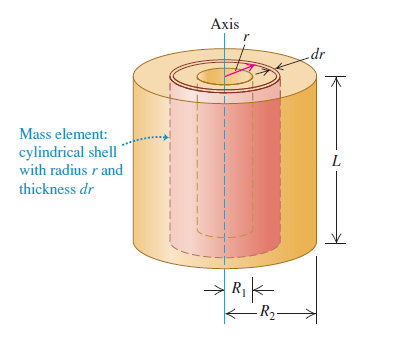
**Hints:** No external torque is exerted on the spinning figure skater, and the axis of rotation is also an axis of symmetry. For a thin hollow cylinder . A slender rod of mass and length rotating about an axis through its center has moment of inertia about this axis: (you don’t have to demonstrate this result).

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**Solution:**

(a)

We consider as infinitesimal volume the volume between the cylinder of radius and : . The infinitesimal mass in this volume is:



The moment of inertia about the axis of the hollow cylinder is:

The volume of the hollow cylinder is:

The mass of the hollow cylinder is:

We obtain,

The moment of inertia is then:

If needed I reminds you: thus

For a thin hollow cylinder of radius , , we obtain:

(b)

Initial moment of inertia about the axis of rotation:

Final moment of inertia about the axis of rotation:

The axis of rotation is axis of symmetry so, the initial angular momentum about a point of the axis of rotation are described by ( and are the initial and final angular velocity vectors):

There is no external torque, so the angular momentum about a point is constant, we obtain :

**Ex. 3.**

A glider of mass is oscillating in SHM on an air track (friction ignored) with an amplitude (horizontal spring mass system, with spring coefficient ). The elastic potential energy is zero at the equilibrium position . You slow it so that its amplitude is halved (the amplitude of the SHM is then ). What happens to its (a) period, frequency, and angular frequency; (b) total mechanical energy; (c) maximum speed; (describe the ratio where is the maximum velocity for the SHM at amplitude and is the maximum velocity for the SHM at amplitude ) (d) velocity at (describe the ratio at where is the velocity at for the original SHM and is the velocity at at for the SHM with amplitude ); (e) potential and kinetic energy at (i.e. describe the ratios and where the and are the potential energy and kinetic energy for the original SHM, and and are the potential energy and kinetic energy for the SHM of amplitude ).

**Solution:**

a)

For a simple harmonic motion corresponding to a glider in horizontal motion, the angular frequency is:

It don’t depends to the amplitude, so the angular frequency don’t change if the amplitude change.

The period is:

The period don’t change when the amplitude change.

The frequency is:

The period frequency don’t change when the amplitude change.

(b)

The total mechanical energy is constant during the SHM (the friction is ignored), when , :

When the amplitude A is halved (), the total mechanical energy decrease of a factor 4:

c) (See demonstration in ppt)

Then,

(d) (See demonstration in ppt)

With the original amplitude

After the amplitude has been halved:

For , we obtain:x,

The ratio between the velocities is:

(e) The kinetic energy is:

The original kinetic energy is:

After the amplitude has been halved, the kinetic energy is:

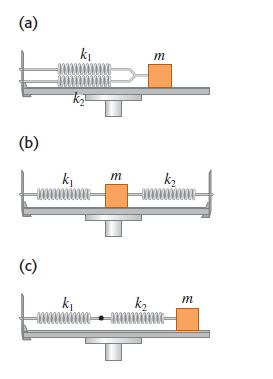
We obtain:

About the elastic potential energy,

It is the same x-position for both cases, so:

**Ex. 4. A combination of two springs**

We consider an association of two springs both attached to a block which exert a spring force for the first spring, and a spring force for the second spring. The block is submitted to a spring force where , is the displacement from equilibrium position of the combination (at equilibrium, both springs are not stretched nor compressed), is the effective force constant for the association of the two springs. Please to describe in respect with and the spring coefficients of the springs, in the three cases described by the figure. About case (c), the spring are considered as massless, and the tension must be the same at any point of the combination, thus .



**Solution:**

1. We consider the displacement from equilibrium position is for the first spring, for the second spring.

In the case a),

And , ,

We obtain:

(b)

Even in this configuration the spring forces are directed toward the same direction, corresponding to the direction toward equilibrium position of each spring (when one spring is compressed, the second spring is stretched). The displacement is , and the previous result is still right:

(c)

The force on the block must be equal to the tension an any point of the spring combination: , and we obtain:

We obtain:

**Ex. 5.**

The displacement of a damped oscillator is described by the equation where the angular frequency is (case of the horizontal motion along the x-axis of a block of mass attached to a massless spring of spring coefficient and submitted to a damping friction force where is the x-velocity of the block, and is the damping constant describing the strength of the damping force). Let the phase angle be zero. (a) According to the equation describing , what is the value of x at . (b) What are the magnitude and direction of the velocity of the block at ? What does the result tell you about the slope of the graph of x versus t near . (c) Obtain an expression for the acceleration at ? (d) Can we really describe the duration as a period of the damped oscillations ? Why is said “pseudo period” rather than a period of the damped oscillatory motion ?

**Solution:**

(a)

(b) There is two possibilities to describe . The first one is to calculate :

The graph of versus t slopes down at . At , ; the graph of versus slopes downs.

(c)

At .

Comment : The sign of depends to and it can be positive, negative or zero.

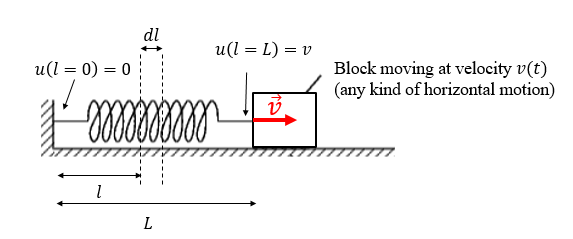
(d) The period of oscillation of a SHM is such as:

But here,

So, . The duration is not a period of the damped oscillations, which is the reason why it is called pseudo-period and the damped oscillations describe a pseudo-periodic motion.

**Ex. 6. A spring with mass.**

Usually, we consider to simplify the description of the SHM the case of a massless spring, i.e. which has no kinetic energy, and which is in an idealized situation. But in a real situation, the spring has a certain mass and a certain kinetic energy we want to describe. We consider the case of the horizontal motion of a spring-mass system where the spring has a mass M. The spring have a length . At at one end of the spring, the velocity is . At the other end of the spring, at , the velocity of the element of infinitesimal mass of the spring is (at this end, a block attached to the spring move at velocity with any possible kind of motion: SHM, damped oscillations, critical damping etc.). For any element of the spring, the velocity at is: . We define the linear mass density of the spring (uniform in all the spring) as follows: . (a) Describe the infinitesimal kinetic energy of the infinitesimal mass at distance from the end in respect to and . (b) Describe the total kinetic energy of the spring in respect to the mass of the spring and the velocity at the end . Why this result is not ?



**Solution:**

(a)

The infinitesimal kinetic energy of the infinitesimal mass at distance from the is:

(b)

The total kinetic energy of the whole spring is:

If the velocity of the infinitesimal mass of the spring was the same in all the spring:

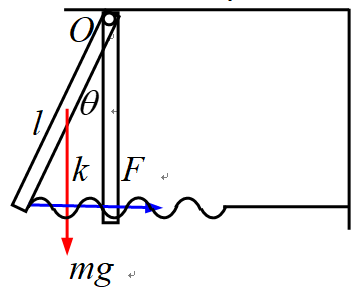
The result about the kinetic energy of the spring is not because the different parts of the spring move at different velocities.

**Comment:** If the spring and the block has the same mass , the kinetic energy of the spring is one third the kinetic energy of the block. It is reasonable to say that when the mass of the spring is much lower than the mass of the block, the kinetic energy of the spring can be ignored and the kinetic energy of the system “block+spring” is then the kinetic energy of the block, as for the case of the ideal massless spring.

**Ex. 7.**

As shown in the figure, a homogeneous thin rod with length *l* and mass *m* is attached to an axis on the ceiling at one end O, while at the other end of the rod it is attached to a spring with spring constant *k*. The angle between the rod and the vertical direction is . There is no friction between the rod and the axis.a) Describe the torque about O of the weight of the rod and the force exerted by the spring on the rod (direction and magnitude in terms of for , in terms of for ). Then, describe the angular acceleration of the rod in terms of and I the moment of inertia of the rod about its axis of rotation. Does this equation describe an angular SHM ? b) Please prove that the system is in simple harmonic motion considering small angles shuch as and shows that the vibrational period of the angular SHM is . Then describe the SHM in terms of its angular amplitude , the time and the angular frequency of the SHM (the origin of time is chosen such as the phase of the SHM is zero).

Help: The moment of inertia of the rod about its axis of rotation is (you don’t have to demonstrate it).



**Solution:**

a)

The torque about a point of a force applied at point P is where

The weight is applied at the center of mass of the rod. Torque about O of the weight is:

We obtain:

Take care that could be positive or negative, because could be positive or negative, depending to the position of the rod and how is chosen the positive direction for the angle (clockwise or anticlockwise )

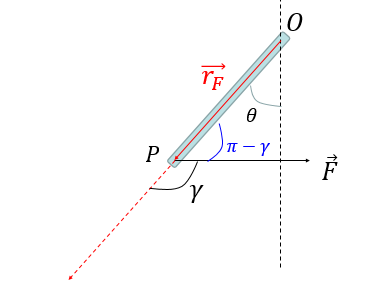
Direction of the torque by the weight (using right-hand rule): perpendicular with the page : if the rod is at the left of the equilibrium position (vertical direction), if the rod is at the right of the equilibrium position)

The force exerted by the spring of the rod is where is directed toward the equilibrium position of the rod. is the x-coordinate of the end of the rod from equilibrium position of the rod ().

The spring force exerted on the rod is applied at the end of the rod. The torque exerted by the spring force on the rod is:

The direction of the torque by about O is the same than the direction of the torque by about O, and its magnitude is:

The angle is named on the following figure:



which is obtained using: , thus and

Thus, the magnitude of the torque by the spring force is:

The rotational equivalent of the Newton’s second law applied on a rigid body in rotation is:

where is the sum of the torques rotating the body, is the moment of inertia about the axis of rotation and is the angular acceleration vector.

We obtain:

This equation doesn’t describe an SHM.

b)

We consider small angles such as and . We obtain:

This equation describes an angular SHM.

Using , we obtain:

And

where

The period of the angular SHM is:

The angular SHM is described by:

But the origin of the time is chosen such as the phase angle equals zero, thus:

Take care that the angular frequency (which is the also the angular velocity associated with the reference circle of the SHM) the angular velocity of the rod .

Here, there is two kinds of angles, which are different: the angle and the angle , and there are two kinds of angular velocity which are also different, the angular velocity and .